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General Matter Coupled $\mathcal{N} = 4$ Gauged Supergravity in Five Dimensions

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Abstract

We construct the general form of matter coupled $\mathcal{N} = 4$ gauged supergravity in five dimensions. Depending on the structure of the gauge group, these theories are found to involve vector and/or tensor multiplets. When self-dual tensor fields are present, they must be charged under a one-dimensional Abelian group and cannot transform non-trivially under any other part of the gauge group. A short analysis of the possible ground states of the different types of theories is given. It is found that AdS ground states are only possible when the gauge group is a direct product of a one-dimensional Abelian group and a semi-simple group. In the purely Abelian, as well as in the purely semi-simple gauging, at most Minkowski ground states are possible. The existence of such Minkowski ground states could be proven in the compact Abelian case.

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1 Introduction

The last few years have witnessed a renewed intense interest in five-dimensional gauged supergravity theories. This interest was largely driven by the study of the AdS/CFT correspondence [1, 2, 3, 4], but also by recent attempts to construct a supersymmetric version of the Randall-Sundrum (RS) scenario [5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

One especially fruitful direction in the study of the AdS/CFT correspondence was its generalization to include certain four-dimensional quantum field theories with non-trivial renormalization group (RG) flows. The best-studied examples of such RG-flows arise from relevant perturbations of the $d = 4, \mathcal{N} = 4$ super Yang-Mills (SYM) theory, and were mapped to domain wall solutions of $d = 5, \mathcal{N} = 8$ gauged supergravity (see [15, 16, 17] for the first explicit examples).

In [17], an RG-flow interpolating between two supersymmetric conformal fixed points of a mass deformed version of $d = 4, \mathcal{N} = 4$ SYM was studied. The corresponding domain wall solution was constructed after a truncation of the $d = 5, \mathcal{N} = 8$ supergravity theory to a particular $\mathcal{N} = 4$ subsector. This subsector describes $\mathcal{N} = 4$ supergravity coupled to two $\mathcal{N} = 4$ tensor multiplets with scalar manifold $\mathcal{M} = SO(1,1)\times SO(5,2)/[SO(5)\times SO(2)]$ and gauge group $SU(2) \times U(1)$. In the same paper this particular $\mathcal{N} = 4$ theory was also conjectured to be the holographic dual of the common sector of all $d = 4, \mathcal{N} = 2$ superconformal gauge theories based on "quiver" diagrams [18, 19]. The holographic duals of these quiver gauge theories were identified in refs. [20, 19] as IIB string theory on $AdS_5 \times (S^5/\Gamma)$, where Γ is a discrete subgroup of $SU(2) \subset SU(4)$ of the ADE type.

The Kaluza-Klein spectrum of IIB string theory on $AdS_5\times(S^5/\Gamma)$ was studied in ref. [21], where the IIB supergravity modes were fit into $SU(2, 2|2)$ multiplets, and in ref. [22], which also considered the $\Gamma = \mathbb{Z}_n$-twisted string states. These twisted states live on $AdS_5 \times S^1$, and were found to correspond to five-dimensional, $\mathcal{N} = 4$ "self-dual" tensor multiplets of $SU(2, 2|2)$ [22].

It has therefore been suggested [23, 17] that five-dimensional, $\mathcal{N} = 4$ gauged supergravity coupled to vector and/or tensor multiplets might encode some non-trivial information on the $\mathcal{N} = 2$ quiver theories even though a *ten*-dimensional supergravity description of the corresponding orbifold theory is not available. In particular, it has been conjectured in [23] that certain aspects of the flow from the $S^5/\mathbb{Z}_2$ to the $T^{1,1}$ compactification of IIB string theory [24] might be captured by a 1/4 BPS domain wall solution of a suitable $\mathcal{N} = 4$ gauged supergravity theory with tensor multiplets. The tensor multiplets are crucial for such a flow because they host the supergravity states dual to the twisted gauge theory operators that drive this flow [23].

Unfortunately, too little was known about five-dimensional, $\mathcal{N} = 4$ gauged supergravity coupled to an arbitrary number of matter (i.e., vector- and tensor-) multiplets in order to further investigate the corresponding gravity description. In fact, only a $SU(2)\times U(1)$ gauging of pure $\mathcal{N} = 4$ supergravity [25] and a peculiar $SU(2)$ gauging of $\mathcal{N} = 4$ supergravity coupled

to vector multiplets [26] were available so far.

It is the purpose of this paper to close this gap in the literature and to construct the general matter-coupled five-dimensional, $\mathcal{N}=4$ gauged supergravity.

This, with [27, 28, 29, 30] for the $\mathcal{N}=2$ case and [31, 32] for the $\mathcal{N}=8$ case, completes also the description of all standard gauged supergravity theories in $d=5$.

These theories should also help to clarify whether the different no-go theorems that have been put forward against a *smooth* supersymmetrization of the RS scenario [6, 7, 8, 33, 14] can be extended to the $\mathcal{N}=4$ sector as well. As for the (more successful) supersymmetrizations based on singular brane sources [9, 10, 11, 12, 13], the theories constructed in this paper allow for the possibility to confine $\mathcal{N}=2$ supergravity theories on the brane, as it is the self-dual tensor fields in the bulk that were shown to circumvent the "no photons on the brane" theorems [34, 35].

The paper is organized as follows. All the gauged $\mathcal{N}=4$ supergravity theories we will construct can be derived from the ungauged Maxwell/Einstein supergravity theories (MESGT) studied in [26]. Section 2.1, therefore, first recalls the relevant properties of these ungauged theories. In Section 2.2, then, we will take a closer look at the global symmetries of these theories and analyze to what extent these global symmetries can be turned into local gauge symmetries. This discussion will reveal some interesting differences to the $\mathcal{N}=2$ and the $\mathcal{N}=8$ cases and also results in a rather natural way to organize the rest of the paper: Section 3 discusses the general gauging of an Abelian group, which turns out to require the introduction of tensor multiplets. In Section 4, we will then construct the combined gauging of a semi-simple group and an Abelian group. The resulting scalar potentials are then briefly analyzed in Section 5, which also contains the reductions to various relevant special cases previously considered in the literature. In the last section, we draw some conclusions and list a few open problems.

2 Ungauged Maxwell/Einstein supergravity

2.1 General setup

The starting point of our construction is the ungauged MESGT of ref. [26] which describes the coupling of n vector multiplets to $\mathcal{N}=4$ supergravity[4]. Our spacetime conventions coincide with those of ref. [26] and are further explained in Appendix A.

The field content of the $\mathcal{N}=4$ supergravity multiplet is

$$\Big(\ e_\mu{}^m\ ,\ \psi^i_\mu\ ,\ A^{ij}_\mu\ ,\ a_\mu\ ,\ \chi^i\ ,\ \sigma\ \Big), \tag{2.1}$$

i.e., it contains one graviton $e_\mu{}^m$, four gravitini ψ^i_μ, six vector fields (A^{ij}_μ, a_μ), four spin 1/2 fermions χ^i and one real scalar field σ. Here, μ/m are the five-dimensional Einstein/Lorentz

[4] For ungauged five-dimensional supergravity theories, vector and tensor fields are Poincaré dual, and we therefore do not have to distinguish between vector and tensor multiplets at this level.

indices and the indices $i, j = 1, \ldots, 4$ correspond to the fundamental representation of the R-symmetry group $USp(4)$ of the underlying $\mathcal{N} = 4$ Poincaré superalgebra. The vector field a_μ is $USp(4)$ inert, whereas the vector fields A_μ^{ij} transform in the $\mathbf{5}$ of $USp(4)$, i.e.,

$$A_\mu^{ij} = -A_\mu^{ji} , \qquad A_\mu^{ij} \Omega_{ij} = 0, \tag{2.2}$$

with Ω_{ij} being the symplectic metric of $USp(4)$. In the following, we will sometimes make use of the local isomorphism $SO(5) \cong USp(4)$ to denote $SO(5)$ representations using $USp(4)$ indices.

An $\mathcal{N} = 4$ vector multiplet is given by

$$\Big(A_\mu \, , \ \lambda^i \, , \ \phi^{ij} \Big), \tag{2.3}$$

where A_μ is a vector field, λ^i denotes four spin 1/2 fields, and the ϕ^{ij} are scalar fields transforming in the $\mathbf{5}$ of $USp(4)$.

Coupling n vector multiplets to supergravity, the field content of the theory can be summarized as follows

$$\Big(e_\mu{}^m \, , \ \psi_\mu^i \, , \ A_\mu^{\tilde{I}} , \ a_\mu \, , \ \chi^i \, , \ \lambda^{ia} \, , \ \sigma \, , \ \phi^x \Big). \tag{2.4}$$

Here, $a = 1, \ldots, n$ counts the number of vector multiplets whereas $\tilde{I} = 1, \ldots, (5+n)$ collectively denotes the A_μ^{ij} and the vector fields of the vector multiplets. Similarly, $x = 1, \ldots, 5n$ is a collective index for the scalar fields in the vector multiplets. We will further adopt the following convention to raise and lower $USp(4)$ indices:

$$T^i = \Omega^{ij} T_j \, , \quad T_i = T^j \Omega_{ji}, \tag{2.5}$$

whereas a, b are raised and lowered with δ^{ab}.

As was shown in [26], the manifold spanned by the $(5n+1)$ scalar fields is

$$\mathcal{M} = \frac{SO(5,n)}{SO(5) \times SO(n)} \times SO(1,1), \tag{2.6}$$

where the $SO(1,1)$ part corresponds to the $USp(4)$-singlet σ of the supergravity multiplet. The theory has therefore a *global* symmetry group $SO(5,n) \times SO(1,1)$ and a *local composite* $SO(5) \times SO(n)$ invariance.

The coset part of the scalar manifold $\mathcal{M}$ can be described in two ways: one can either, as in (2.4), choose a parameterization in terms of coordinates ϕ^x, where the metric g_{xy} on the coset part of $\mathcal{M}$ is given in terms of $SO(5) \times SO(n)$ vielbeins by (cf. Appendix B):

$$g_{xy} = \frac{1}{4} f_x^{ija} f_{yij}^a, \tag{2.7}$$

such that the kinetic term for the scalar fields takes the typical form $\frac{1}{2} g_{xy} \partial_\mu \phi^x \partial^\mu \phi^y$ of a nonlinear sigma-model. This way of describing $\mathcal{M}$ is particularly useful for discussing geometrical properties of the theory.

As an alternative description, one can use coset representatives $L_{\tilde{I}}{}^A$ where $\tilde{I}$ denotes a $\mathcal{G} = SO(5,n)$ index, and $A = (ij, a)$ is a $\mathcal{H} = SO(5) \times SO(n)$ index. Denoting the inverse of $L_{\tilde{I}}{}^A$ by $L_A{}^{\tilde{I}}$ (i.e., $L_{\tilde{I}}{}^A \, L_B{}^{\tilde{I}} \; = \delta^A_B$), the vielbeins on $\mathcal{G}/\mathcal{H}$ and the composite $\mathcal{H}$-connections are determined from the 1-form:

$$L^{-1} dL \;=\; Q^{ab}\, \mathfrak{T}_{ab} + Q^{ij}\, \mathfrak{T}_{ij} + P^{aij}\, \mathfrak{T}_{aij}, \tag{2.8}$$

where $(\mathfrak{T}_{ab}, \mathfrak{T}_{ij})$ are the generators of the Lie algebra $\mathfrak{h}$ of $\mathcal{H}$, and $\mathfrak{T}_{aij}$ denotes the generators of the coset part of the Lie algebra $\mathfrak{g}$ of $\mathcal{G}$. More precisely,

$$Q^{ab} \;=\; L^{\tilde{I}a} dL_{\tilde{I}}{}^b \qquad \text{and} \qquad Q^{ij} \;=\; L^{\tilde{I}ik} dL_{\tilde{I}k}{}^j \tag{2.9}$$

are the composite $SO(n)$ and $USp(4)$ connections, respectively, and

$$P^{aij} \;=\; L^{\tilde{I}a} dL_{\tilde{I}}{}^{ij} = -\frac{1}{2} f^{aij}_x \, d\phi^x \tag{2.10}$$

describes the space-time pull-back of the $\mathcal{G}/\mathcal{H}$ vielbein. Note that Q^{ab}_μ is antisymmetric in the $SO(n)$ indices, whereas Q^{ij}_μ is symmetric in i and j. This second way of parameterizing the scalar manifold is particularly useful to exhibit the action of the different invariance groups as clearly as possible. We will make this choice in what follows for the construction of the gauged theories. Eqs. (2.10) and (B.8)-(B.11) can be used to easily switch between both formalisms. Appendix B contains more details on the geometry of $\mathcal{G}/\mathcal{H}$.

The Lagrangian of the ungauged MESGT reads [26]:

$$\begin{aligned}
e^{-1}\,\mathcal{L} \;=\; & -\frac{1}{2}R - \frac{1}{2}\,\bar{\psi}^i_\mu\,\Gamma^{\mu\nu\rho}\,D_\nu\psi_{\rho i} - \frac{1}{4}\Sigma^2\,a_{\tilde{I}\tilde{J}}\,F^{\tilde{I}}_{\mu\nu}F^{\mu\nu\tilde{J}} - \frac{1}{4}\Sigma^{-4}\,G_{\mu\nu}G^{\mu\nu} \\
& -\frac{1}{2}\,(\partial_\mu\sigma)^2 - \frac{1}{2}\,\bar{\chi}^i\,\not{D}\chi_i - \frac{1}{2}\bar{\lambda}^{ia}\,\not{D}\lambda^a_i - \frac{1}{2}\,P^{aij}_\mu P^\mu_{aij} \\
& -\frac{i}{2}\bar{\chi}^i\Gamma^\mu\Gamma^\nu\psi_{\mu i}\,\partial_\nu\sigma + i\bar{\lambda}^{ia}\,\Gamma^\mu\Gamma^\nu\psi^j_\mu\,P_{\nu ij}{}^a \\
& + \frac{\sqrt{3}}{6}\Sigma L^{ij}_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\bar{\chi}_i\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu j} - \frac{1}{4}\Sigma L^a_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\bar{\lambda}^{ai}\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu i} \\
& - \frac{1}{2\sqrt{6}}\Sigma^{-2}\,\bar{\chi}^i\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu i}G_{\rho\sigma} + \frac{5i}{24\sqrt{2}}\Sigma^{-2}\bar{\chi}^i\Gamma^{\rho\sigma}\chi_i G_{\rho\sigma} \\
& - \frac{i}{12}\Sigma L^{ij}_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\bar{\chi}_i\,\Gamma^{\rho\sigma}\chi_j - \frac{i}{2\sqrt{3}}\Sigma L^a_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\,\bar{\lambda}^{ia}\Gamma^{\rho\sigma}\chi_i - \frac{i}{8\sqrt{2}}\Sigma^{-2}G_{\rho\sigma}\bar{\lambda}^{ia}\Gamma^{\rho\sigma}\lambda^a_i \\
& + \frac{i}{4}\Sigma L^{ij}_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\,\bar{\lambda}^a_i\Gamma^{\rho\sigma}\lambda^a_j - \frac{i}{4}\Sigma L^{ij}_{\tilde{I}}\,F^{\tilde{I}}_{\rho\sigma}\,[\bar{\psi}_{\mu i}\Gamma^{\mu\nu\rho\sigma}\psi_{\nu j} + 2\,\bar{\psi}^\rho_i\,\psi^\sigma_j] \\
& - \frac{i}{8\sqrt{2}}\,\Sigma^{-2}G_{\rho\sigma}\,[\bar{\psi}^i_\mu\Gamma^{\mu\nu\rho\sigma}\psi_{\nu i} + 2\bar{\psi}^{\rho i}\,\psi^\sigma_i] \\
& + \frac{\sqrt{2}}{8}e^{-1}\,C_{\tilde{I}\tilde{J}}\,\epsilon^{\mu\nu\rho\sigma\lambda}\,F^{\tilde{I}}_{\mu\nu}F^{\tilde{J}}_{\rho\sigma}\,a_\lambda + e^{-1}\mathcal{L}_{4f},
\end{aligned} \tag{2.11}$$

and the supersymmetry transformation laws are given by[5]

$$
\begin{aligned}
\delta e_\mu^m &= \frac{1}{2}\bar{\varepsilon}^i\Gamma^m\psi_{\mu i} \\
\delta\psi_{\mu i} &= D_\mu\varepsilon_i + \frac{i}{6}\Sigma L_{\tilde{I}ij}F^{\tilde{I}}_{\rho\sigma}(\Gamma_\mu^{\ \rho\sigma} - 4\delta_\mu^\rho\Gamma^\sigma)\varepsilon^j \\
&\quad + \frac{i}{12\sqrt{2}}\Sigma^{-2}G_{\rho\sigma}(\Gamma_\mu^{\ \rho\sigma} - 4\delta_\mu^\rho\Gamma^\sigma)\varepsilon_i + \text{3-fermion terms} \\
\delta\chi_i &= -\frac{i}{2}\partial\!\!\!/\sigma\varepsilon_i + \frac{\sqrt{3}}{6}\Sigma L_{\tilde{I}ij}F^{\tilde{I}}_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon^j - \frac{1}{2\sqrt{6}}\Sigma^{-2}G_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon_i \\
\delta\lambda_i^a &= iP^a_{\mu ij}\Gamma^\mu\varepsilon^j - \frac{1}{4}\Sigma\, L^a_{\tilde{I}}F^{\tilde{I}}_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon_i + \text{3-fermion terms} \\
\delta A_\mu^{\tilde{I}} &= \vartheta_\mu^{\tilde{I}} \\
\delta a_\mu &= \frac{1}{\sqrt{6}}\Sigma^2\bar{\varepsilon}^i\Gamma_\mu\chi_i - \frac{i}{2\sqrt{2}}\Sigma^2\bar{\varepsilon}^i\psi_{\mu i} \\
\delta\sigma &= \frac{i}{2}\bar{\varepsilon}^i\chi_i \\
\delta L^{ij}_{\tilde{I}} &= -iL^a_{\tilde{I}}(\delta_k^{[i}\delta_l^{j]} - \frac{1}{4}\Omega^{ij}\Omega_{kl})\bar{\varepsilon}^k\lambda^{la} \\
\delta L^a_{\tilde{I}} &= -iL_{\tilde{I}ij}\bar{\varepsilon}^i\lambda^{ja}
\end{aligned} \tag{2.12}
$$

with

$$
\vartheta_\mu^{\tilde{I}} \equiv -\frac{1}{\sqrt{3}}\Sigma^{-1}L^{\tilde{I}}_{ij}\bar{\varepsilon}^i\Gamma_\mu\chi^j - i\Sigma^{-1}L^{\tilde{I}}_{ij}\bar{\varepsilon}^i\psi_\mu^j + \frac{1}{2}L^{\tilde{I}}_a\Sigma^{-1}\bar{\varepsilon}^i\Gamma_\mu\lambda_i^a. \tag{2.13}
$$

Here,

$$
F^{\tilde{I}}_{\mu\nu} = (\partial_\mu A^{\tilde{I}}_\nu - \partial_\nu A^{\tilde{I}}_\mu)\ , \quad G_{\mu\nu} = (\partial_\mu a_\nu - \partial_\nu a_\mu)\ , \tag{2.14}
$$

are the Abelian field strengths of the vector fields, whereas the scalar field in the supergravity multiplet is parameterized by

$$
\Sigma = e^{\frac{1}{\sqrt{3}}\sigma}. \tag{2.15}
$$

Moreover, the covariant derivative, D_μ, that acts on the fermions is given by

$$
D_\mu\lambda_i^a = \nabla_\mu\lambda_i^a + Q_{\mu i}{}^j\lambda_j^a + Q_\mu^{ab}\lambda_i^b, \tag{2.16}
$$

with ∇_μ being the Lorentz- and spacetime covariant derivative.

Supersymmetry imposes

$$
a_{\tilde{I}\tilde{J}} = L^{ij}_{\tilde{I}}L_{\tilde{J}ij} + L^a_{\tilde{I}}L^a_{\tilde{J}}\ , \quad C_{\tilde{I}\tilde{J}} = L^{ij}_{\tilde{I}}L_{\tilde{J}ij} - L^a_{\tilde{I}}L^a_{\tilde{J}}, \tag{2.17}
$$

where $a_{\tilde{I}\tilde{J}}$ acts as a metric on the $\tilde{I}, \tilde{J}$ indices:

$$
L^A_{\tilde{I}} = a_{\tilde{I}\tilde{J}}\,L^{\tilde{J}A}. \tag{2.18}
$$

[5]We use the following (anti) symmetrization symbols:$(ij) \equiv \frac{1}{2}(ij + ji)$, $[ij] \equiv \frac{1}{2}(ij - ji)$.

The symmetric tensor $C_{\tilde{I}\tilde{J}}$ turns out to be *constant*, and in fact is nothing but the $SO(5,n)$ metric.

Supersymmetry also imposes a number of additional algebraic and differential relations on the $L^{A}_{\tilde{I}}$, which are listed in Appendix B. Actually, the requirement of invariance under supersymmetry heavily constrains the form of the possible couplings and, as we show in Appendix C using the superspace language, if one does not consider non-minimal couplings, those presented here already use all the freedom allowed by the $\mathcal{N}=4$ superalgebra.

2.2 The global symmetries and their possible gaugings

The above ungauged Maxwell/Einstein supergravity theories are subject to several global and local invariances. Apart from supersymmetry, the *local* invariances are[6]

- a local composite $USp(4)\times SO(n)$ symmetry
- a Maxwell-type invariance of the form[7]

$$\begin{aligned} A^{\tilde{I}}_{\mu} &\longrightarrow A^{\tilde{I}}_{\mu}+\partial_{\mu}\Lambda^{\tilde{I}} && (2.19)\\ a_{\mu} &\longrightarrow a_{\mu}+\partial_{\mu}\Lambda. && (2.20)\end{aligned}$$

In addition, the ungauged theories of Section 2.1 are invariant under *global* $SO(1,1)\times SO(5,n)$ transformations, which exclusively act on the vector fields, the coset representatives $L^{\tilde{I}}_{A}$, and the scalar field $\Sigma=e^{\frac{\sigma}{\sqrt{3}}}$ according to

$$\begin{aligned} \delta_{SO(5,n)}A^{\tilde{I}}_{\mu} &= \alpha^{r}M_{(r)\tilde{J}}^{\tilde{I}}A^{\tilde{J}}_{\mu}\\ \delta_{SO(5,n)}L^{\tilde{I}}_{A} &= \alpha^{r}M_{(r)\tilde{J}}^{\tilde{I}}L^{\tilde{J}}_{A}\\ \delta_{SO(5,n)}\Sigma &= 0\\ \delta_{SO(5,n)}a_{\mu} &= 0 \end{aligned}\qquad (2.21)$$

and

$$\begin{aligned} \delta_{SO(1,1)}A^{\tilde{I}}_{\mu} &= -\lambda A^{\tilde{I}}_{\mu}\\ \delta_{SO(1,1)}L^{\tilde{I}}_{A} &= 0\\ \delta_{SO(1,1)}\Sigma &= \lambda\Sigma\\ \delta_{SO(1,1)}a_{\mu} &= 2\lambda a_{\mu}, \end{aligned}\qquad (2.22)$$

[6]In large parts of the literature on "gauged" supergravity, none of these two local invariances is referred to as a "gauge" symmetry: the local composite $Usp(4)\times SO(n)$ symmetry is not based on physical vector fields, and the Maxwell-type invariance is, in this context, more viewed as a special case of a more general invariance under $C^{(p)}\longrightarrow C^{(p)}+d\Lambda^{(p-1)}$ for arbitrary p-form fields $C^{(p)}$ and $(p-1)$-forms $\Lambda^{(p-1)}$. The term "gauged" supergravity, by contrast, is used when some physical vector fields of a (usually $\mathcal{N}$-extended) supergravity theory couple to other matter fields via gauge covariant derivatives. In most cases, the gauge symmetry of such a theory reduces to a global symmetry of an "ungauged" supergravity theory when the gauge coupling is turned off.

[7]Note that it is essential for this symmetry to hold that the $C_{\tilde{I}\tilde{J}}$ be constant.

where $M_{(r)\tilde{J}}^{\tilde{I}} \in \mathfrak{so}(5,n)$, and α^r $(r = 1, \ldots, \dim(SO(5,n)))$ and λ are some infinitesimal parameters.

We will now analyze to what extent this *global* $SO(1,1) \times SO(5,n)$ invariance can be turned into a *local* (i.e., Yang-Mills-type) gauge symmetry by introducing minimal couplings of some of the vector fields, a process commonly referred to as "gauging".

The first thing to notice is that the $SO(1,1)$ factor can*not* be gauged, as *all* the vector fields transform non-trivially under it (i.e., none of the vector fields in the theory could serve as the corresponding (neutral) Abelian gauge field). Thus, we can restrict ourselves to gaugings of subgroups $K \subset SO(5,n)$.

The vector fields $A_\mu^{\tilde{I}}$ and the coset representatives $L_A^{\tilde{I}}$ transform in the $(\mathbf{5}+\mathbf{n})$ of $SO(5,n)$; all other fields are $SO(5,n)$-inert (cf. eqs. (2.21)). Hence, any gauge group $K \subset SO(5,n)$ has to act non-trivially on at least some of the vector fields $A_\mu^{\tilde{I}}$ (as well as on some of the coset representatives $L_A^{\tilde{I}}$).

Having physical applications in mind, we will restrict our subsequent discussion to gauge groups K that are either

(i) Abelian or

(ii) semi-simple or

(iii) a direct product of a semi-simple and an Abelian group.

2.2.1 K is Abelian

Let us first assume that K is Abelian. As we mentioned above, K has to act non-trivially on at least some of the vector fields $A_\mu^{\tilde{I}}$. In addition to such non-singlets of K, there might also be vector fields among the $A_\mu^{\tilde{I}}$ that are K-inert. Thus, in general, we have a decomposition of the form

$$(\mathbf{5}+\mathbf{n})_{SO(5,n)} \longrightarrow \text{singlets}(K) \oplus \text{non-singlets}(K). \tag{2.23}$$

Let us split the vector fields $A_\mu^{\tilde{I}}$ accordingly:

$$A_\mu^{\tilde{I}} = (A_\mu^I, A_\mu^M)$$

where the indices $I, J, \ldots$ label the K-singlets, and $M, N, \ldots$ denote the non-singlets.

As was first pointed out in [31, 32] for the $\mathcal{N} = 8$ theory, the presence of such non-singlet vector fields poses a problem for the supersymmetric gauging of K and requires the dualization of the charged vectors into self-dual [36] tensor fields.

As we will explain in Section 3, this conversion of the A_μ^M to "self-dual" tensor fields $B_{\mu\nu}^M$ is achieved in practice by simply replacing all field strengths $F_{\mu\nu}^M$ by $B_{\mu\nu}^M$ and by adding a kinetic term of the form $\mathcal{L}_{BdB} = B \wedge dB$ to the Lagrangian. The derivative in $\mathcal{L}_{BdB}$ then

turns out to be automatically K-covariantized by the $B \wedge B \wedge a$-term in the Lagrangian, which originates from the $F \wedge F \wedge a$ term in the ungauged Lagrangian (2.11) upon the replacement $F_{\mu\nu}^{M} \longrightarrow B_{\mu\nu}^{M}$. This has an important consequence: only the $SO(5,n)$ singlet vector field a_{μ} can couple to the tensor fields $B_{\mu\nu}^{M}$ for this kind of gauging, and, consequently, any Abelian gauge group K can be at most one-dimensional, i.e., it can be either $SO(2)$ or $SO(1,1)$ with gauge vector a_{μ}. Note that the converse is also true: If tensor fields have to be introduced in order to gauge a (not necessarily Abelian) subgroup $K \subset SO(5,n)$, these tensor fields can only be charged with respect to a one-dimensional Abelian subgroup of K. This is an interesting difference to the $\mathcal{N}=8$ and the $\mathcal{N}=2$ theories, where the tensor fields can also transform in nontrivial representations of non-Abelian gauge groups K [31, 32, 28].

We will discuss the gauging of an Abelian group K in Section 3.

2.2.2 K is semi-simple

Let us now come to the case when the gauge group $K \subset SO(5,n)$ is semi-simple. In that case, some of the vector fields $A_{\mu}^{\tilde{I}}$ of the ungauged theory have to transform in the adjoint representation of K so that they can be promoted to the corresponding Yang-Mills-type gauge fields. Put another way, the $(\mathbf{5}+\mathbf{n})$ of $SO(5,n)$ has to contain the adjoint of K as a sub-representation. A priori, one would therefore expect the decomposition

$$(\mathbf{5}+\mathbf{n})_{SO(5,n)} \longrightarrow \text{adjoint}(K) \oplus \text{singlets}(K) \oplus \text{non-singlets}(K). \tag{2.24}$$

Just as for the Abelian case, any non-singlet vector fields outside the adjoint of K would have to be converted to "self-dual" tensor fields $B_{\mu\nu}^{M}$. On the other hand, we already found that, due to the peculiar structure of the Chern-Simons term in the ungauged theory (2.11), only the vector field a_{μ} could possibly couple to such tensor fields. As one vector field alone can never gauge a semi-simple group, we are led to the conclusion that the gauging of a semi-simple group K can *not* introduce any tensor fields, and we can restrict ourselves to the case

$$(\mathbf{5}+\mathbf{n})_{SO(5,n)} \longrightarrow \text{adjoint}(K) \oplus \text{singlets}(K). \tag{2.25}$$

The vector fields in the adjoint will then serve as the K gauge fields, whereas the singlets are mere "spectator" vector fields.

Let us conclude this subsection with a rough classification of the possible *compact* semi-simple gauge groups K. Obviously, a compact gauge group K has to be a subgroup of the maximal compact subgroup, $SO(5) \times SO(n) \subset SO(5,n)$. Furthermore, K can be either a subgroup of the $SO(5)$ factor or a subgroup of the $SO(n)$ factor or a direct product of both of these.

If $K \subset SO(5)$, it can only be the standard $SO(3) \cong SU(2)$ subgroup of $SO(5)$, because this is the only semi-simple subgroup of $SO(5)$ with the property that the $\mathbf{5}$ of $SO(5)$ decomposes according to (2.25).

If $K \subset SO(n)$, the fundamental representation of $SO(n)$ has to contain the adjoint of K (as well as possibly some K-singlets). However, the adjoint of *any* compact semi-simple group K can be embedded into the fundamental representation of $SO(n)$ as long as $\dim(K) \leq n$ (One simply has to take the positive definite Cartan-Killing metric as the $SO(\dim(K))$ metric). Thus, *any* compact semi-simple group K can be gauged along the above lines as long as $\dim(K) \leq n$.

An obvious combination of the previous two paragraphs finally covers the case when K is a direct product of an $SO(5)$- and an $SO(n)$-subgroup.

We will not write down the gauging of a semi-simple group separately, as it can be easily obtained as a special case of the combined Abelian and semi-simple gauging, which we discuss now.

2.2.3 $K = K_{\textbf{Abelian}} \times K_{\textbf{semi-simple}}$

If K is a direct product of a semi-simple and an Abelian group, one simply has to combine Sections 2.2.1 and 2.2.2:

Let K_S denote the semi-simple and K_A the Abelian part of K. Then the gauging of K_A implies

$$(\mathbf{5}+\mathbf{n})_{SO(5,n)} \longrightarrow \text{singlets}(K_A) \oplus \text{non-singlets}(K_A). \tag{2.26}$$

As discussed in Section 2.2.1, the non-singlet vector fields of K_A have to be converted to self-dual tensor fields. These tensor fields cannot be charged under the semi-simple part K_S (see the discussion of Section 2.2.2). Hence, K_S can only act on the singlets of K_A. Furthermore, the action of K_S on these K_A singlets cannot introduce additional tensor fields, i.e., there can be no non-singlets of K_S beyond the adjoint of K_S:

$$\text{singlets}(K_A) \longrightarrow \text{adjoint}(K_S) \oplus \text{singlets}(K_S). \tag{2.27}$$

As described in Section 2.2.1, the Chern-Simons term of the ungauged theory (2.11) requires K_A to be one-dimensional, i.e., we can have either $K = U(1) \times K_S$ or $K = SO(1,1) \times K_S$.

We will take a closer look at this general gauging in Section 4.

3 The gauging of an Abelian group $K_A \subset SO(5,n)$

The goal of this paper is to construct the most general gauging of $\mathcal{N}=4$ supergravity coupled to an arbitrary number of vector and tensor multiplets. As was discussed in the previous section, this corresponds to the simultaneous gauging of a one-dimensional Abelian group K_A and a semi-simple group K_S. However, it is the Abelian gauging that introduces the tensor fields, so it is worth treating this technically more subtle case separately:

The gauging of an Abelian group $K_A \subset SO(5,n)$ proceeds in three steps (see also the $\mathcal{N}=2$ [28] and $\mathcal{N}=8$ [31, 32] cases):

Step 1:
As discussed in Section 2.2.1, the most general decomposition of the $(\mathbf{5}+\mathbf{n})$ of $SO(5,n)$ with respect to an Abelian gauge group K_A is

$$(\mathbf{5}+\mathbf{n})_{SO(5,n)} \longrightarrow \text{singlets}(K_A) \oplus \text{non-singlets}(K_A), \tag{3.1}$$

We will again use $I, J, \ldots$ for the K_A singlets and $M, N, \ldots$ for the non-singlets of K_A. Note that the (rigid) K_A invariance of the Chern Simons term in (2.11) already implies that we have $C_{IM} = 0$: If $C_{IM} \neq 0$, we would need $\Lambda_N^M C_{IM} = 0$ for the invariance of $C_{IM} F^I \wedge F^M \wedge a$, where Λ_N^M denotes the K_A transformation matrix of the non-singlet vector fields A_μ^M. But then the K_A representation space spanned by the A_μ^M would have to have one non-trivial null-eigenvector, i.e., there would be at least one singlet among the A_μ^M, contrary to our assumption. This same condition $C_{IM} = 0$ also follows by requiring the closure of the supersymmetry algebra on the vector fields A_μ^I, which in the superspace analysis amounts to the requirement of the closure of the Bianchi Identities for the supercurvatures F^I.

In order to gauge K_A, the non-singlet vector fields A_μ^M now have to be converted to "self-dual" tensor fields $B_{\mu\nu}^M$, whereas the singlet vector fields A_μ^I will play the rôle of spectator vector fields. Using the Nœther procedure, the tensor field "dualization" is done by first literally replacing all $F_{\mu\nu}^M$ in the ungauged Lagrangian (2.11) and the ungauged transformation laws (2.12) by tensor fields $B_{\mu\nu}^M$. This is more than a mere relabeling, as the $B_{\mu\nu}^M$ are no longer assumed to be the curls of vector fields A_μ^M. Because of this, we no longer have a Bianchi identity for the tensor fields $B_{\mu\nu}^M$, i.e., in general, we now have $dB^M \neq 0$. This, however, already breaks supersymmetry, because some of the supersymmetry variations in the ungauged theory only vanished due to $dF^M = 0$. As a remedy, one therefore adds the extra term

$$\mathcal{L}_{BdB} = \frac{1}{4g_A} \epsilon^{\mu\nu\rho\sigma\lambda} \Omega_{MN} B_{\mu\nu}^M \partial_\rho B_{\sigma\lambda}^N \tag{3.2}$$

to the Lagrangian and requires the supersymmetry transformation rule of the $B_{\mu\nu}^M$ to be

$$\begin{aligned} \delta B_{\mu\nu}^M &= 2\partial_{[\mu}\vartheta_{\nu]}^M + 2g_A a_{[\mu}\Lambda_N^M \vartheta_{\nu]}^N - g_A \Sigma L_{Nij} \Omega^{NM} \bar{\psi}_{[\mu}^i \Gamma_{\nu]} \varepsilon^j \\ &\quad + \frac{i}{4} g_A \Sigma L_N^a \Omega^{NM} \bar{\lambda}^{ia} \Gamma_{\mu\nu} \varepsilon_i - \frac{i}{2\sqrt{3}} g_A \Sigma L_{Nij} \Omega^{NM} \bar{\chi}^i \Gamma_{\mu\nu} \varepsilon^j. \end{aligned} \tag{3.3}$$

Here, g_A denotes the K_A coupling constant, Ω_{MN} is a constant and antisymmetric metric with inverse Ω^{MN}:

$$\Omega_{MN}\Omega^{NP} = \delta_M^P, \qquad \Omega_{MN} = -\Omega_{NM},$$

and ϑ_μ^M is as defined in eq. (2.13). It is not too hard to show that inserting $\delta B_{\mu\nu}^M$ into $\mathcal{L}_{BdB}$ exactly cancels the supersymmetry breaking terms that arise due to $dB^M \neq 0$.

Step 2:
Closer inspection of $\mathcal{L}_{BdB}$ reveals that it combines with

$$\mathcal{L}_{BBa} = \frac{\sqrt{2}}{8} C_{MN} \epsilon^{\mu\nu\rho\sigma\lambda} B^M_{\mu\nu} B^N_{\rho\sigma} a_\lambda$$

(which stems from the former Chern-Simons term in (2.11)), to automatically form a K_A-covariant derivative:

$$\mathcal{L}_{BdB} + \mathcal{L}_{BBa} = \frac{1}{4g_A} \epsilon^{\mu\nu\rho\sigma\lambda} \Omega_{MN} B^M_{\mu\nu} \mathfrak{D}_\rho B^N_{\sigma\lambda},$$

where

$$\mathfrak{D}_\rho B^M_{\mu\nu} \equiv \nabla_\rho B^M_{\mu\nu} + g_A a_\rho \Lambda^M_N B^N_{\mu\nu} \tag{3.4}$$

with

$$\Lambda^M_N = \frac{1}{\sqrt{2}} \Omega^{MP} C_{PN} \tag{3.5}$$

being the K_A-transformation matrix of the tensor fields. The same is true for the first two terms in $\delta B^M_{\mu\nu}$ (eq. (3.3)), which naturally combine to $2\mathfrak{D}_{[\mu}\vartheta^M_{\nu]}$. However, apart from these two derivatives, none of the various other derivatives in the Lagrangian or the transformation laws are covariantized with respect to K_A. On the other hand, the only derivatives affected by this are derivatives acting on the coset representatives L^A_M, L^M_A, which exclusively appear inside the composite connections $Q_{\mu i}{}^j$, $Q_{\mu a}{}^b$ and the $P^a_{\mu ij}$ (cf. eqs. (2.9), (2.10)). Introducing K_A-covariant derivatives

$$\mathfrak{D}_\rho L^M_A \equiv \partial_\rho L^M_A + g_A a_\rho \Lambda^M_N L^N_A \tag{3.6}$$

also inside $Q_{\mu i}{}^j$, $Q_{\mu a}{}^b$ and $P^a_{\mu ij}$ is tantamount to making the replacements

$$\begin{aligned}
Q_{\mu i}{}^j &\longrightarrow \mathcal{Q}_{\mu i}{}^j = Q_{\mu i}{}^j - g_A a_\mu \Lambda^M_N L_{Mik} L^{Nkj} && (3.7)\\
Q_{\mu a}{}^b &\longrightarrow \mathcal{Q}_{\mu a}{}^b = Q_{\mu a}{}^b + g_A a_\mu \Lambda^M_N L_{Ma} L^{Nb} && (3.8)\\
P^a_{\mu ij} &\longrightarrow \mathcal{P}^a_{\mu ij} = P^a_{\mu ij} - g_A a_\mu \Lambda^M_N L_{Mij} L^{Na} && (3.9)
\end{aligned}$$

everywhere in the Lagrangian and the transformation laws.

Step 3:
After Step 1 and Step 2, the theory is now K_A gauge invariant to all orders in g_A and supersymmetric to order $(g_A)^0$. However, at order $(g_A)^{\geq 1}$ the theory fails to be supersymmetric due to the numerous new g_A-dependent terms we introduced. As a third step, one therefore has to restore supersymmetry by adding a few g_A-dependent (but gauge invariant) terms to the covariantized Lagrangian and transformation rules.

After all these modifications, the final Lagrangian is given by (up to 4-Fermion terms):

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\;& -\frac{1}{2}R - \frac{1}{2}\,\bar{\psi}^i_\mu\,\Gamma^{\mu\nu\rho}\,\mathcal{D}_\nu\psi_{\rho i} - \frac{1}{4}\Sigma^2\, a_{\tilde{I}\tilde{J}}\, H^{\tilde{I}}_{\mu\nu} H^{\mu\nu\tilde{J}} - \frac{1}{4}\Sigma^{-4}\, G_{\mu\nu}G^{\mu\nu} \\
& -\frac{1}{2}\,(\partial_\mu\sigma)^2 - \frac{1}{2}\,\bar{\chi}^i\,\not{\mathcal{D}}\chi_i - \frac{1}{2}\bar{\lambda}^{ia}\not{\mathcal{D}}\lambda^a_i - \frac{1}{2}\,\mathcal{P}^{aij}_\mu\mathcal{P}^\mu_{aij} \\
& -\frac{i}{2}\bar{\chi}^i\Gamma^\mu\Gamma^\nu\psi_{\mu i}\,\partial_\nu\sigma + i\bar{\lambda}^{ia}\,\Gamma^\mu\Gamma^\nu\psi^j_\mu\,\mathcal{P}_{\nu ij}{}^a \\
& +\frac{\sqrt{3}}{6}\Sigma L^{ij}_{\tilde{I}}\,H^{\tilde{I}}_{\rho\sigma}\bar{\chi}_i\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu j} - \frac{1}{4}\Sigma L^a_{\tilde{I}}\,H^{\tilde{I}}_{\rho\sigma}\bar{\lambda}^{ai}\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu i} \\
& -\frac{1}{2\sqrt{6}}\Sigma^{-2}\,\bar{\chi}^i\,\Gamma^\mu\Gamma^{\rho\sigma}\psi_{\mu i}G_{\rho\sigma} + \frac{5i}{24\sqrt{2}}\Sigma^{-2}\bar{\chi}^i\Gamma^{\rho\sigma}\chi_i G_{\rho\sigma} \\
& -\frac{i}{12}\Sigma L^{ij}_{\tilde{I}}H^{\tilde{I}}_{\rho\sigma}\bar{\chi}_i\,\Gamma^{\rho\sigma}\chi_j - \frac{i}{2\sqrt{3}}\Sigma L^a_{\tilde{I}}\,H^{\tilde{I}}_{\rho\sigma}\bar{\lambda}^{ia}\Gamma^{\rho\sigma}\chi_i - \frac{i}{8\sqrt{2}}\Sigma^{-2}G_{\rho\sigma}\bar{\lambda}^{ia}\Gamma^{\rho\sigma}\lambda^a_i \\
& +\frac{i}{4}\Sigma L^{ij}_{\tilde{I}}H^{\tilde{I}}_{\rho\sigma}\bar{\lambda}^a_i\Gamma^{\rho\sigma}\lambda^a_j - \frac{i}{4}\Sigma L^{ij}_{\tilde{I}}H^{\tilde{I}}_{\rho\sigma}\,[\bar{\psi}_{\mu i}\Gamma^{\mu\nu\rho\sigma}\psi_{\nu j} + 2\,\bar{\psi}^\rho_i\,\psi^\sigma_j] \\
& -\frac{i}{8\sqrt{2}}\,\Sigma^{-2}G_{\rho\sigma}\,[\bar{\psi}^i_\mu\Gamma^{\mu\nu\rho\sigma}\psi_{\nu i} + 2\bar{\psi}^{\rho i}\,\psi^\sigma_i] \\
& +\frac{\sqrt{2}}{8}e^{-1}\,C_{IJ}\,\epsilon^{\mu\nu\rho\sigma\lambda}\,F^I_{\mu\nu}F^J_{\rho\sigma}\,a_\lambda + \frac{e^{-1}}{4g_A}\epsilon^{\mu\nu\rho\sigma\lambda}\Omega_{MN}B^M_{\mu\nu}\mathfrak{D}_\rho B^N_{\sigma\lambda} \\
& +\frac{3i}{2}g_A U_{ij}\bar{\psi}^i_\mu\Gamma^{\mu\nu}\psi^j_\nu + ig_A N_{ijab}\bar{\lambda}^{ia}\lambda^{jb} - \frac{5i}{2}g_A U_{ij}\bar{\chi}^i\chi^j \\
& -g_A V^a_{ij}\bar{\psi}^i_\mu\Gamma^\mu\lambda^{ja} - 2\sqrt{3}g_A U_{ij}\bar{\psi}^i_\mu\Gamma^\mu\chi^j - \frac{4i}{\sqrt{3}}g_A V^a_{ij}\bar{\chi}^i\lambda^{ja} - g_A^2\mathcal{V}^{(A)},
\end{aligned}
\tag{3.10}
$$

and the supersymmetry transformation laws are given by (up to 3-fermion terms)

$$
\begin{aligned}
\delta e^m_\mu \;=\;& \frac{1}{2}\bar{\varepsilon}^i\Gamma^m\psi_{\mu i} \\
\delta\psi_{\mu i} \;=\;& \mathcal{D}_\mu\varepsilon_i + \frac{i}{6}\Sigma L_{\tilde{I}ij}H^{\tilde{I}}_{\rho\sigma}(\Gamma_\mu{}^{\rho\sigma} - 4\delta^\rho_\mu\Gamma^\sigma)\varepsilon^j \\
& +\frac{i}{12\sqrt{2}}\Sigma^{-2}G_{\rho\sigma}(\Gamma_\mu{}^{\rho\sigma} - 4\delta^\rho_\mu\Gamma^\sigma)\varepsilon_i + ig_A U_{ij}\Gamma_\mu\varepsilon^j \\
\delta\chi_i \;=\;& -\frac{i}{2}\not{\partial}\sigma\varepsilon_i + \frac{\sqrt{3}}{6}\Sigma L_{\tilde{I}ij}H^{\tilde{I}}_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon^j - \frac{1}{2\sqrt{6}}\Sigma^{-2}G_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon_i - 2\sqrt{3}g_A U_{ij}\varepsilon^j \\
\delta\lambda^a_i \;=\;& i\mathcal{P}^a_{\mu ij}\Gamma^\mu\varepsilon^j - \frac{1}{4}\,\Sigma L^a_{\tilde{I}}H^{\tilde{I}}_{\rho\sigma}\Gamma^{\rho\sigma}\varepsilon_i + g_A V^a_{ij}\varepsilon^j \\
\delta A^I_\mu \;=\;& \vartheta^I_\mu \\
\delta B^M_{\mu\nu} \;=\;& 2\mathfrak{D}_{[\mu}\vartheta^M_{\nu]} - g_A\Sigma L_{Nij}\Omega^{NM}\bar{\psi}^i_{[\mu}\Gamma_{\nu]}\varepsilon^j \\
& +\frac{i}{4}g_A\Sigma L^a_N\Omega^{NM}\bar{\lambda}^{ia}\Gamma_{\mu\nu}\varepsilon_i - \frac{i}{2\sqrt{3}}g_A\Sigma L_{Nij}\Omega^{NM}\bar{\chi}^i\Gamma_{\mu\nu}\varepsilon^j \\
\delta a_\mu \;=\;& \frac{1}{\sqrt{6}}\Sigma^2\bar{\varepsilon}^i\Gamma_\mu\chi_i - \frac{i}{2\sqrt{2}}\Sigma^2\bar{\varepsilon}^i\psi_{\mu i} \\
\delta\sigma \;=\;& \frac{i}{2}\bar{\varepsilon}^i\chi_i
\end{aligned}
$$

$$\begin{aligned}
\delta L^{ij}_{\tilde{I}} &= -iL^a_{\tilde{I}}(\delta^{[i}_k\delta^{j]}_l - \frac{1}{4}\Omega^{ij}\Omega_{kl})\bar{\varepsilon}^k\lambda^{la} \\
\delta L^a_{\tilde{I}} &= -iL_{\tilde{I}ij}\bar{\varepsilon}^i\lambda^{ja}. \qquad (3.11)
\end{aligned}$$

In the above expressions, we have introduced the tensorial quantity

$$H^{\tilde{I}}_{\mu\nu} \equiv (F^I_{\mu\nu}, B^M_{\mu\nu}) \qquad (3.12)$$

as well as the new $USp(4) \times SO(n)$ covariant derivatives

$$\mathcal{D}_\mu\lambda^a_i = \nabla_\mu\lambda^a_i + \mathcal{Q}_{\mu i}{}^j\lambda^a_j + \mathcal{Q}^{ab}_\mu\lambda^b_i \qquad (3.13)$$

with the new, g_A-dependent connections (3.7) and (3.8).

The new scalar field dependent quantities U_{ij}, V^a_{ij}, N_{ijab}, as well as the scalar potential $\mathcal{V}^{(A)}$ are fixed by supersymmetry:

$$\begin{aligned}
U_{ij} &= \frac{\sqrt{2}}{6}\Sigma^2\Lambda^N_M L_{N(i|k|}L^{Mk}{}_{j)} \qquad (3.14) \\
V^a_{ij} &= \frac{1}{\sqrt{2}}\Sigma^2\Lambda^N_M L_{Nij}L^{Ma} \qquad (3.15) \\
N_{ijab} &= -\frac{1}{2\sqrt{2}}\Sigma^2\Lambda^N_M L^a_N L^{Mb}\Omega_{ij} + \frac{3}{2}U_{ij}\delta_{ab} \qquad (3.16) \\
\mathcal{V}^{(A)} &= \frac{1}{2}V^a_{ij}V^{aij}. \qquad (3.17)
\end{aligned}$$

In particular, the U_{ij} and V^a_{ij} shifts in the supersymmetry transformations of the fermionic fields are all expressed in terms of the so-called "boosted structure constants", which are just the representation matrices of the vector fields under the gauged group multiplied by the coset representatives. This is a common feature of all gauged supergravities which have a scalar manifold given by an ordinary homogeneous space [37].

4 The simultaneous gauging of $K_S \times K_A$

The starting point for the simultaneous gauging of $K_S \times K_A$ is the K_A gauged Lagrangian (3.10) with the corresponding supersymmetry variations (3.11). In this case, fields carrying an $SO(5,n)$ index $\tilde{I}$ decompose according to (2.26) and (2.27) into adjoint(K_S), singlets(K_S) and non-singlets(K_A) fields. However, to avoid the introduction of a third type of $SO(5,n)$ index beside I and M, we will collectively denote the adjoint(K_S) and singlet(K_S) fields by I, J, K, while non-singlet(K_A) fields will carry indices M, N. The actual distinction between the adjoint(K_S) and the singlet(K_S) fields will be made implicitly by the K_S structure constant f^K_{IJ} which will be taken to vanish whenever one index denotes a K_S singlet field. In order for the Chern-Simons term to be globally K_S invariant, these structure constants have to satisfy the following identity

$$C_{IJ}\,f^I_{KL} + C_{IL}\,f^I_{KJ} \;=\; 0. \qquad (4.1)$$

The additional gauging of K_S essentially proceeds in two steps:

Step 1:
All derivatives acting on K_S charged fields must be K_S covariantized. This modifies the definition of the field strengths of the gauge fields A_μ^I in the standard way:

$$\begin{array}{rcl} F^I_{\mu\nu} & \longrightarrow & \mathcal{F}^I_{\mu\nu} \;=\; F^I_{\mu\nu} + g_S\, A^J_\mu f^I_{JK} A^K_\nu \\ \text{thus}\quad H^{\tilde{I}}_{\mu\nu} & \longrightarrow & \mathcal{H}^{\tilde{I}}_{\mu\nu} \;=\; (\mathcal{F}^I_{\mu\nu}, B^M_{\mu\nu}). \end{array} \tag{4.2}$$

In order to simplify the notation, we use the same derivative symbols as in the previous section, but now also with g_S dependent contributions:

$$\mathfrak{D}_\mu L_A^{\tilde{I}} \;=\; \partial_\mu L_A^{\tilde{I}} + g_A \delta_M^{\tilde{I}} a_\mu \Lambda_N^M L_A^N + g_S \delta_I^{\tilde{I}} A_\mu^J f_{JK}^I L_A^K. \tag{4.3}$$

This in turns modifies the $USp(4)$ and $SO(n)$ connections, as well as the vielbein $\mathcal{P}^a_{\mu ij}$, as these quantities contain derivatives of the coset representatives. Again, we use the same symbols as in the previous section, but now also include the new g_S dependent contributions, i.e.,

$$\mathcal{Q}_{\mu i}{}^j \;=\; Q_{\mu i}{}^j - g_A a_\mu \Lambda_N^M L_{Mik} L^{Nkj} + g_S A_\mu^J L_{ik}^K f_{JK}^I L_I^{kj}, \tag{4.4}$$

$$\mathcal{Q}_{\mu a}{}^b \;=\; Q_{\mu a}{}^b + g_A a_\mu \Lambda_N^M L_{Ma} L^{Nb} - g_S A_\mu^J L_a^K f_{JK}^I L_I^b, \tag{4.5}$$

$$\mathcal{P}^a_{\mu ij} \;=\; P^a_{\mu ij} - g_A a_\mu \Lambda_N^M L_{Mij} L^{Na} - g_S A_\mu^J L_{ij}^K f_{JK}^I L_I^a. \tag{4.6}$$

These new objects appear in the derivatives of the fermions, which means that

$$\mathcal{D}_\mu \lambda_i^a = \nabla_\mu \lambda_i^a + \mathcal{Q}_{\mu i}{}^j \lambda_j^a + \mathcal{Q}_\mu^{ab} \lambda_i^b \tag{4.7}$$

should now be understood as containing also the g_S dependent terms inside the composite connections (4.4)-(4.5).

After these modifications in the supersymmetry transformations rules (3.11) and the Lagrangian (3.10), the latter is supersymmetric up to a small number of g_S dependent terms. These uncanceled terms take the form

$$\begin{array}{rcl} e^{-1}\,\delta(\mathcal{L}) & = & \frac{1}{2} g_S \mathcal{F}^J_{\rho\sigma} f^I_{JK} L_{Iik} L^{Kkj} \bar{\psi}^i_\mu \Gamma^{\mu\rho\sigma} \varepsilon_j + \frac{i}{2} g_S \mathcal{F}^J_{\rho\sigma} f^I_{JK} L^K_{ij} L^a_I \bar{\lambda}^{ia} \Gamma^{\rho\sigma} \varepsilon^j \\ & & + g_S f^K_{JI} L^{Iij} L^a_K \mathcal{P}^{\mu a}_{ij} (\delta A^J_\mu). \end{array} \tag{4.8}$$

Step 2:
Just as in the Abelian case, the remaining terms can be compensated by adding fermionic mass terms as well as potential terms to the Lagrangian, and by suitable modifications of the fermionic transformation rules.

The additional mass and potential terms needed are

$$\begin{array}{rcl} \mathcal{L}_{\text{mass}} & = & \frac{3i}{2} g_S S_{ij}\, \bar{\psi}^i_\mu \Gamma^{\mu\nu} \psi^j_\nu + i g_S I_{ijab}\, \bar{\lambda}^{ia} \lambda^{jb} + \frac{i}{2} g_S S_{ij}\, \bar{\chi}^i \chi^j + g_S T^a_{ij}\, \bar{\psi}^i_\mu \Gamma^\mu \lambda^{ja} \\ & & + \sqrt{3} g_S S_{ij}\, \bar{\psi}^i_\mu \Gamma^\mu \chi^j - \frac{2i}{\sqrt{3}} g_S T^a_{ij}\, \bar{\chi}^i \lambda^{ja} - g_S g_A \mathcal{V}^{(AS)} - g_S^2 \mathcal{V}^{(S)}, \end{array} \tag{4.9}$$

whereas the additional pieces in the transformation rules of the fermions take the form

$$\delta_{\text{new}}\psi_{\mu i} = i g_S S_{ij}\Gamma_\mu \varepsilon^j, \tag{4.10}$$
$$\delta_{\text{new}}\lambda^a_i = g_S T^a_{ij}\varepsilon^j, \tag{4.11}$$
$$\delta_{\text{new}}\chi_i = g_S\sqrt{3}\, S_{ij}\varepsilon^j. \tag{4.12}$$

The scalar field dependent functions in the above expressions are given by

$$S_{ij} = -\frac{2}{9}\Sigma^{-1} L^J_{(i|k|} f^K_{JI} L^{kl}_K L^I_{|l|j)}, \tag{4.13}$$
$$T^a_{ij} = -\Sigma^{-1} L^{Ja} L^{Kk}_{(i} f^I_{JK} L_{I|k|j)}, \tag{4.14}$$
$$I_{ijab} = -\frac{3}{2} S_{ij}\delta_{ab} - \Sigma^{-1} L^{Ja} L^K_{ij} f^I_{JK} L^b_I, \tag{4.15}$$

whereas the potential terms read

$$\mathcal{V}^{(AS)} = -18\, U_{ij}\, S^{ij}\ , \quad \mathcal{V}^{(S)} = -\frac{9}{2}\, S_{ij}S^{ij} + \frac{1}{2}\, T^a_{ij}\, T^{aij}. \tag{4.16}$$

In addition, supersymmetry implies a series of derivative relations on the scalar quantities introduced above, which will be very useful for the study of the vacua of the theory:

$$\mathcal{D}_\mu U_{ij} = \frac{2}{\sqrt{3}}\,\partial_\mu\sigma\, U_{ij} - \frac{2}{3}\, V^a_{(i|k|}\,\mathcal{P}^{ak}_{\mu\ \ j)}, \tag{4.17}$$
$$\mathcal{D}_\mu V^a_{ij} = 2\, N_{[i}{}^k{}_{ab}\,\mathcal{P}^b_{\mu|k|j]} - 3U_{[i|k|}\,\mathcal{P}^{ak}_{\mu\ \ j]} + \frac{2}{\sqrt{3}}\, V^a_{ij}\,\partial_\mu\sigma, \tag{4.18}$$
$$\mathcal{D}_\mu S_{ij} = \frac{2}{3}\, T^a_{(i|k|}\mathcal{P}^{ak}_{\mu\ \ j)} - \frac{1}{\sqrt{3}}\, S_{ij}\partial_\mu\sigma, \tag{4.19}$$
$$\mathcal{D}_\mu T^a_{ij} = -\frac{1}{\sqrt{3}}\,\partial_\mu\sigma\, T^a_{ij} + 3\, S_{(i|k|}\mathcal{P}^{ak}_{\mu\ \ j)} - 2\, I_{(i|k|ab}\mathcal{P}^{bk}_{\mu\ \ j)}, \tag{4.20}$$

where the derivatives should be understood as being fully $SO(n)$ and $USp(4)$ covariant derivatives based on the new composite connections and vielbeins (4.4)-(4.6). Using the numerous constraints satisfied by the coset representatives $L^A_{\tilde I}$, one can show that eqs. (4.17)-(4.20) follow automatically from the definitions of the fermionic shifts.

Just as for the gauging of the Abelian factor, we point out that the shifts in the supersymmetry laws of the fermionic fields are given by the boosted structure constants of the gauged semi-simple group.

5 The scalar potential $\mathcal{V}$

The full scalar potential obtained from the $(K_S \times K_A)$ gauging is

$$\begin{aligned}\mathcal{V} &= g_A^2\mathcal{V}^{(A)} + g_A g_S\mathcal{V}^{(AS)} + g_S^2\mathcal{V}^{(S)}\\ &= \frac{1}{2}\Big[\, g_A^2\, V^a_{ij}V^{aij} - 36\, g_A g_S\, U_{ij}S^{ij} + g_S^2\left(T^a_{ij}T^{aij} - 9\, S_{ij}S^{ij}\right)\Big].\end{aligned} \tag{5.1}$$

This potential could have been derived directly from the expression of the shifts in the supersymmetry laws of the fermionic fields and the expression of their kinetic terms. Indeed, as proved for the first time in four dimensions [38], in all supersymmetric theories the potential is given by squaring the shifts of the fermions using the metric defined by the kinetic terms:

$$-\frac{1}{2}\,\delta_{\varepsilon_1}\bar{\psi}^i_\mu\,\Gamma^{\mu\nu\rho}\,\delta_{\varepsilon_2}\psi_{\rho i}-\frac{1}{2}\,\delta_{\varepsilon_1}\bar{\chi}^i\,\Gamma^\nu\delta_{\varepsilon_2}\chi_i-\frac{1}{2}\delta_{\varepsilon_1}\bar{\lambda}^{ia}\Gamma^\nu\delta_{\varepsilon_2}\lambda^a_i=\frac{1}{4}\bar{\varepsilon}^i_1\Gamma^\nu\varepsilon_{2i}\mathcal{V}, \tag{5.2}$$

provided one satisfies the generalized Ward-identity:

$$\begin{aligned}\frac{1}{4}\Omega_{ij}\,\mathcal{V} &= \frac{1}{2}g_A^2 V_i^{ak}V^a_{kj}+g_A g_S\Big[9(S_i{}^kU_{kj}+U_i{}^kS_{kj})+\frac{1}{2}(V_i^{ak}T^a_{kj}-T_i^{ak}V^a_{kj})\Big]\\ &\quad -\frac{1}{2}g_S^2\Big[T_i^{ak}T^a_{kj}-9S_i{}^kS_{kj}\Big].\end{aligned} \tag{5.3}$$

This can indeed be verified by using the expression of these objects in terms of the coset representatives and then using the properties of the latter.

It is instructive to rewrite the scalar potential, as well as the other scalar field dependent quantities in terms of the Killing vectors characterizing the gauged isometries of the scalar manifold $\mathcal{M}$. To this end, we switch back to the parameterization in terms of the ϕ^x (cf. Sect. 2.1). On the ϕ^x, $K_S\times K_A$ transformations act as isometries:

$$\delta\phi^x \;=\; \alpha K^x+\alpha^I K^x_I, \tag{5.4}$$

where α and α^I are, respectively, the infinitesimal K_A and K_S transformation parameters, whereas K^x and K^x_I denote the corresponding Killing vectors, which can be expressed as:

$$K^x \;=\; \frac{1}{2}\,f^{xija}\,\Lambda_N^M\,L_{Mij}L^{Na}\;,\quad K^x_I \;=\; \frac{1}{2}\,f^{xija}\,L^J_{ij}f^K_{IJ}L^a_K. \tag{5.5}$$

As in the $\mathcal{N}=2$ case [30, 39], one has Killing prepotentials defined as:

$$D_x\mathcal{P}^{(A)}_{ij} \;=\; K^y\,\mathcal{R}_{xyij}\;,\quad D_x\mathcal{P}^{(S)}_{Iij} \;=\; K^y_I\,\mathcal{R}_{xyij}, \tag{5.6}$$

where $\mathcal{R}_{xyij}$ is the $USp(4)$ curvature and the derivatives D_x contain the original composite connections of the *ungauged* theory (see eq. (2.9)):

$$Q^{ab}_x \;=\; L^{\tilde{I}a}\partial_x L_{\tilde{I}}{}^b \qquad\text{and}\qquad Q^{ij}_x \;=\; L^{\tilde{I}ik}\partial_x L_{\tilde{I}k}{}^j. \tag{5.7}$$

These prepotentials are found to be:

$$\mathcal{P}^{(A)}_{ij} \;=\; -\frac{1}{2}\,\Lambda_N^M\,L^N_{ik}L_M{}^k_j, \qquad \mathcal{P}^{(S)}_{Iij} \;=\; \frac{1}{2}\,f^K_{IJ}L_i^{Jk}\,L_{Kkj} \tag{5.8}$$

and are exactly the objects that appear in the shift of the composite connection:

$$\mathcal{Q}_{\mu i}{}^j=Q_{\mu i}{}^j-2g_A a_\mu\mathcal{P}^{(A)}{}_i{}^j-2g_S A^I_\mu\mathcal{P}^{(S)}_I{}_i{}^j. \tag{5.9}$$

Moreover, the prepotentials have to satisfy the algebra of the K_S isometries [39]

$$K^x_I K^y_J\,\mathcal{R}_{xyij}+4\mathcal{P}^{(S)}_{[I}{}_i{}^k\mathcal{P}^{(S)}_{J]kj}+f^K_{IJ}\mathcal{P}^{(S)}_{Kij} \;=\; 0. \tag{5.10}$$

However, unlike in the $\mathcal{N}=2$ case [39, 30], this does not give any additional constraint on the prepotentials. Indeed, using eqs. (5.5), (5.8) and (B.6), one can show that the relation is identically satisfied.

Now, using the above relations, the shifts in the gaugini transformation rules can be expressed as:

$$V^a_{ij} \;=\; \frac{1}{2\sqrt{2}}\,\Sigma^2\, f^a_{xij}\, K^x\;, \qquad T^a_{ij} \;=\; \frac{1}{2}\,\Sigma^{-1}\, L_i^{Ik}\, f^a_{xkj}\, K^x_I, \tag{5.11}$$

whereas the shifts in the gravitini transformation rules are given in terms of the prepotentials:

$$U_{ij} \;=\; \frac{\sqrt{2}}{3}\,\Sigma^2\, \mathcal{P}^{(A)}_{ij}\;, \qquad S_{ij} \;=\; -\frac{4}{9}\,\Sigma^{-1}\, L_i^{Ik}\, \mathcal{P}^{(S)}_{Ikj}. \tag{5.12}$$

A general study of the vacua and domain-wall solutions of this theory is beyond the scope of this paper, and will be left for a future publication [40]. Instead, we conclude with a list of special cases, some of which were already studied in the literature.

- $g_S = 0$:

  Turning off the K_S gauging leads us back to the Abelian theories studied in Sect. 3. In this case, the potential reduces to[8]

  $$\mathcal{V} \;=\; g_A^2\, \mathcal{V}^{(A)} \;=\; \frac{1}{2}\, g_A^2\, V^a_{ij} V^{aij} \;=\; \frac{1}{2}\, g_A^2\, \Sigma^4\, \widehat{V}^a_{ij} \widehat{V}^{aij}, \tag{5.13}$$

  where, in the following, a hat always denotes the σ independent part of the scalar field dependent quantities. It is then easy to see that minimizing the potential with respect to σ requires the potential to vanish at the extrema:

  $$\partial_\sigma \mathcal{V}|_{\phi_c} \;=\; 0 \qquad \Leftrightarrow \qquad \mathcal{V}|_{\phi_c} \;=\; 0. \tag{5.14}$$

  Hence, if this potential admits critical points, they have to correspond to Minkowski ground states of the theory. On the other hand, such critical points do not necessarily have to exist. In the case when K_A is *compact* (i.e., $K_A = U(1)$), however, one can prove that the theory indeed has at least one Minkowski ground state: expressing the potential in terms of the Killing vectors, we obtain, using (5.11):

  $$\mathcal{V} \;=\; \frac{1}{4}\, g_A^2\, \Sigma^4\, g_{xy}\, K^x K^y. \tag{5.15}$$

  Being compact, the $U(1)$ gauge symmetry is a subgroup of the maximal compact subgroup $\mathcal{H} = SO(5)\times SO(n)$ of $\mathcal{G} = SO(5,n)$, i.e., it is a subgroup of the isotropy group of the scalar manifold. This ensures that there exists at least one point $\phi_o \in \mathcal{M}$ that is invariant under the action of K_A, i.e., at this point, the $U(1)$ Killing vector and, consequently, the potential vanish:

  $$K^x|_{\phi_o} = 0 \quad \Longrightarrow \quad \mathcal{V}|_{\phi_o} = 0. \tag{5.16}$$

  Hence, there is a least one Minkowski ground state for the $U(1)$ gauged theory.

[8]Note the similarity with the corresponding $\mathcal{N}=2$ theories [28], where the part of the scalar potential that is due to the presence of tensor multiplets is also non-negative.

- $g_A = 0$:

  At a first look, a naive limit $g_A \to 0$ seems to be ill-defined due to the presence of the $\frac{1}{g_A}$ term in the Lagrangian (3.10). As was shown in [41], however, there is a perfectly well-defined procedure to take this limit, based on a field redefinition of the form

$$B^M_{\mu\nu} \;\longrightarrow\; g_A\, C^M_{\mu\nu} + F^M_{\mu\nu}, \tag{5.17}$$

  where $F^M_{\mu\nu}$ is the curl of some vector field A^M_μ. After such a limit has been taken, the $K_S \times K_A$ theory reduces to a theory of the type discussed in Sect. 2.2.2, in which only a semi-simple group K_S is gauged.

  The scalar potential of such a theory is then given by:

$$\mathcal{V} \;=\; g_S^2\, \mathcal{V}^{(S)} \;=\; \frac{1}{2}\, g_S^2 \left(T^a_{ij} T^{aij} - 9\, S_{ij} S^{ij}\right). \tag{5.18}$$

  Again, one can factor out the σ-dependent part, and write the potential as

$$\mathcal{V} \;=\; \frac{1}{2}\, g_S^2\, \Sigma^{-2} \left(\widehat{T}^a_{ij} \widehat{T}^{aij} - 9\, \widehat{S}_{ij} \widehat{S}^{ij}\right). \tag{5.19}$$

  The form of the σ-dependence shows that, as in the purely Abelian case, at most Minkowski ground states can exist:

$$\partial_\sigma \mathcal{V}|_{\phi_c} \;=\; 0 \qquad \Leftrightarrow \qquad \mathcal{V}|_{\phi_c} \;=\; 0. \tag{5.20}$$

  However, in contrast to the Abelian case, we cannot prove the existence of such a vacuum state, even if we assume K_S to be compact. Indeed, whereas the T^a_{ij} are expressed in terms of the Killing vectors as in eq. (5.11), the S_{ij} are proportional to the Killing prepotentials, which need not vanish at the invariant point $\phi_o \in \mathcal{M}$ where the Killing vectors are zero for a compact K_S. Eq. (5.6) merely implies that the prepotentials should be covariantly constant at that point.

  The particular choice $K_S = SU(2) \subset SO(5)$ corresponds to the case studied in [26]. Indeed, our potential (5.18) has precisely the same form as the potential of ref. [26]. However, in our case, we have

$$\partial_x \mathcal{V} \;\neq\; 0, \tag{5.21}$$

  due to the fact that we allowed for a general mass term I_{ijab} for the gaugini, that also contains a part antisymmetric in a, b, whereas in [26], one has $I_{ijab} = -\frac{3}{2} S_{ij} \delta_{ab}$. Thus, it seems that the $SU(2)$ gauging considered in [26] is not the most general one.

- $n = 0$:

  In this case, only the $\mathcal{N} = 4$ supergravity multiplet is present. This means that the global symmetry group $SO(1,1) \times SO(5,n)$ of the ungauged theory (2.11) reduces to $SO(1,1) \times SO(5)$, where $SO(5)$ is just the R-symmetry group of the $\mathcal{N} = 4$ Poincaré

superalgebra. As discussed in Section 2.2.2, K_S can then only be the standard $SO(3) \cong SU(2)$ subgroup of $SO(5)$, and we therefore expect to recover the $SU(2) \times U(1)$ gauged theory of [25] when this $SU(2)$ is gauged together with the obvious additional $SO(2) \cong U(1)$ subgroup of $SO(5)$.

Indeed, in the absence of exterior vector multiplets, we have $V^a_{ij} = T^a_{ij} = 0$, whereas the σ independent quantities $\widehat{U}_{ij}$ and $\widehat{S}_{ij}$ become constant matrices ($\widehat{U}_{ij} = \frac{1}{6}(\Gamma_{45})_{ij}$ and $\widehat{S}_{ij} = -\frac{2}{9}(\Gamma_{123})_{ij} = -\frac{2}{9}(\Gamma_{45})_{ij}$)[9] such that the potential (5.1) reduces to

$$\mathcal{V} \;=\; -g_S\left(g_A\,\Sigma + g_S\,\Sigma^{-2}\right), \tag{5.22}$$

where we have discarded some numerical factors. This potential indeed coincides with the potential of [25].

- $n = 2, K = SU(2) \times U(1)$:

  In the ungauged theory with scalar manifold

$$\mathcal{M} \;=\; \frac{SO(5,2)}{SO(5) \times SO(2)} \times SO(1,1), \tag{5.23}$$

  one can gauge an $SU(2) \times U(1)$ subgroup such that four tensor fields appear, two in the supergravity multiplet and two coming from the former vector multiplets. To this end, the $SU(2)$ factor has to be the standard $SO(3) \cong SU(2) \subset SO(5)$, whereas the $U(1)$ factor has to be a diagonal subgroup of the two $SO(2)$'s in the obvious subgroup $SO(2) \times SO(3) \times SO(2) \subset SO(5,2)$. This corresponds to the $\mathcal{N} = 4$ theory obtained in [17] by truncating the $\mathcal{N} = 8$ theory to an $SU(2)_I \subset SU(4)$ invariant subsector. The relevance of this sector for the study of certain RG-flows in the AdS/CFT correspondence was stressed in the Introduction.

6 Conclusions

In this paper, we have studied the possible gaugings of matter coupled $\mathcal{N} = 4$ supergravity in five dimensions. All these theories can be obtained from the ungauged MESGTs of ref. [26] by gauging appropriate subgroups of the global symmetry group $SO(5,n) \times SO(1,1)$, where n is the number of vector plus tensor multiplets. A more careful analysis of the possible gauge groups revealed that the $SO(1,1)$ factor cannot be gauged, whereas the possible gaugeable subgroups K of $SO(5,n)$ naturally fall into three different categories:

If K is Abelian, its gauging requires the dualization of the K charged vector fields into self-dual antisymmetric tensor fields. For consistency with the structure of the ungauged theory, such an Abelian gauge group K has to be one-dimensional, i.e., it can be either $U(1)$ or $SO(1,1)$. In the case $K = U(1)$, we could prove the existence of at least one Minkowski

[9] The gamma matrices introduced here refer to Euclidean gamma matrices of $SO(5)$.

ground state. Such an existence proof could not be given for the case $K = SO(1,1)$ [10]. In any case, however, no AdS ground states are possible if K is Abelian.

The gauging of a semi-simple group $K \subset SO(5,n)$, by contrast, does *not* introduce any tensor fields. Conversely, self-dual tensor fields can only be charged with respect to a one-dimensional Abelian group. This is an interesting difference to the $\mathcal{N} = 2$ and $\mathcal{N} = 8$ cases [31, 32, 28], where the tensor fields could also be charged under a non-Abelian group. As for the critical points of the scalar potential, we found that they can at most correspond to Minkowski space-times. However, we were not able to give an existence proof similar to the $U(1)$ case. To sum up, neither the pure Abelian gauging nor the gauging of a semi-simple group alone allow for the existence of AdS ground states. As for $\mathcal{N} = 4$ supersymmetric AdS ground states, this is to be expected from the corresponding AdS superalgebra $SU(2,2|2)$, which has the R-symmetry group $SU(2) \times U(1)$.

This situation changes when $K = K_S \times K_A$ is the direct product of a semi-simple and an Abelian group. In this case, the Abelian group K_A again has to be one-dimensional, and tensor fields are required for its gauging. The resulting scalar potential for the simultaneous gauging of K_S and K_A consists of the sum of the potentials that already arise in the separate gaugings of K_S and K_A, but also contains an interference term that depends on both coupling constants. Again, this is an interesting difference to the analogous $\mathcal{N} = 2$ gaugings [28], where no such interference was found. Due to all these different contributions to the scalar potential, it is now no longer excluded that AdS ground states exist. In fact, this is to be expected from various special cases that have appeared in the literature [25, 17]. On the other hand, the more complicated form of this potential also makes a thorough analysis of its critical points along the lines of, e.g., [43] more difficult.

At this point, we should mention that the conclusions of our analysis on the possible gaugings might be modified if one were to consider even more general gaugings based on non semi-simple algebras as was done, e.g., in the $\mathcal{N} = 8$ case in [44].

Having obtained the general form of matter coupled $\mathcal{N} = 4$ gauged supergravity, one might now try to clarify several open problems. First, it would be interesting to see whether some of these theories can indeed be used to extract information on the $\mathcal{N} = 2$ quiver gauge theories discussed in [18, 19, 20, 22, 17, 21], and to recover the gravity duals of various RG-flows that have been constructed [17] or were conjectured to exist [24]. Furthermore, one now has the tools to elucidate some of the questions raised in [45] concerning possible alternative gaugings of certain $\mathcal{N} = 2$ matter coupled theories.

Another interesting issue would be to clarify whether the gaugings presented here can be reproduced by dimensional reduction of the heterotic string on a torus with internal fluxes, as was done in [46]. In fact, the dimensional reduction does not introduce any additional tensor fields (beside the NS-NS two-form). As these were shown to play a essential rôle for the Abelian gauging, one might expect some differences concerning this part of the gauging between the two approaches.

[10] In fact, in [42] a counter-example was found for a particular non-compact gauging of the $\mathcal{N} = 2$ analog.

Finally, it should be extremely interesting to use this new theory to obtain new insight on the existence of smooth supersymmetric brane-world solutions à la Randall-Sundrum.

Even if, so far, the study of the $\mathcal{N} = 2$ theory has not yet provided a final answer [47], it has anyhow restricted the analysis to models involving vector- and hypermultiplets. Unfortunately, there are many possible scalar manifolds and gaugings that one can build for such models. In any case, the models that can be obtained by reduction of some $\mathcal{N} = 4$ realization must be contained in the general setup presented here. Moreover, since the $\mathcal{N} = 4$ scalar manifold has the unique form presented in (2.6), the analysis of the possible gaugings and flows, should be simpler and we hope that it should lead to more stringent results.

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Appendix

A Spacetime and gamma matrix conventions

This appendix summarizes our spacetime and gamma matrix conventions. The spacetime metric $g_{\mu\nu}$ and the fünfbein $e_\mu{}^m$ are related by

$$g_{\mu\nu} = e_\mu{}^m e_\nu{}^n \eta_{mn}$$

with $\eta_{mn} = \mathrm{diag}(-++++)$. The indices $\mu, \nu \dots$ and $m, n, \dots$ are 'curved' and 'flat' indices, respectively, and run from 0 to 4. Our conventions regarding the Riemann tensor and its contractions are

$$\begin{aligned} R_{\mu\nu mn}(\omega) &= [(\partial_\mu \omega_{\nu mn} + \omega_{\mu mp}\,\omega_\nu{}^p{}_n) - (\mu \leftrightarrow \nu)], && \text{(A.1)} \\ R(\omega, e) &= e^{\nu m} e^{\mu n} R_{\mu\nu mn}, && \text{(A.2)} \end{aligned}$$

where $\omega_{\mu mn}(e)$ is the spin connection defined via the usual constraint

$$\partial_{[\mu} e_{\nu]m} + \omega_{[\mu m}{}^n e_{\nu]n} \; = \; 0.$$

The gamma matrices Γ_m in five dimensions are constant, complex-valued (4×4)-matrices satisfying

$$\{\Gamma_m, \Gamma_n\} = 2\eta_{mn}.$$

Given a set $\{\Gamma_0, \Gamma_1, \Gamma_2, \Gamma_3\}$ of gamma matrices in *four* dimensions, the definition

$$\Gamma_4 := \pm i \Gamma_0 \Gamma_1 \Gamma_2 \Gamma_3 \qquad \text{(A.3)}$$

yields a representation of the corresponding Clifford algebra in five dimensions for either choice of the sign. We will always choose this sign such that eq. (A.4) below holds. The gamma matrices Γ_μ with a 'curved' index are simply defined by $\Gamma_\mu := e_\mu{}^m \Gamma_m$.

Antisymmetrized products of gamma matrices are denoted by

$$\Gamma_{\mu_1 \mu_2 \ldots \mu_p} := \Gamma_{[\mu_1} \Gamma_{\mu_2} \ldots \Gamma_{\mu_p]},$$

where the square brackets again denote antisymmetrization with "strength one". Using this definition, we have

$$\Gamma^{\mu\nu\rho\sigma\lambda} \;=\; \frac{i}{e}\, \epsilon^{\mu\nu\rho\sigma\lambda}. \tag{A.4}$$

The charge conjugation matrix C in five dimensions satisfies

$$C \Gamma^\mu C^{-1} = (\Gamma^\mu)^T. \tag{A.5}$$

It can be chosen such that

$$C^T = -C = C^{-1}.$$

The antisymmetry of C and the defining property (A.5) imply that the matrix $(C\Gamma_{\mu_1 \ldots \mu_p})$ is symmetric for $p = 2, 3$ and antisymmetric for $p = 0, 1, 4, 5$.

All spinors we consider are anticommuting symplectic Majorana spinors

$$\bar{\chi}^i := (\chi_i)^\dagger \Gamma_0 = \Omega^{ij} \chi_j^T C.$$

As a consequence of the symmetry properties of $(C\Gamma_{\mu_1 \ldots \mu_p})$, these (anticommuting) spinors satisfy the following useful identities:

$$\bar{\Psi}^i \Gamma_{\mu_1 \ldots \mu_p} \chi^j = \begin{cases} +\bar{\chi}^j \Gamma_{\mu_1 \ldots \mu_p} \Psi^i & (p = 0, 1, 4, 5), \\ -\bar{\chi}^j \Gamma_{\mu_1 \ldots \mu_p} \Psi^i & (p = 2, 3). \end{cases}$$

which implies

$$\bar{\Psi}^i \Gamma_{\mu_1 \ldots \mu_p} \chi_i = \begin{cases} -\bar{\chi}^i \Gamma_{\mu_1 \ldots \mu_p} \Psi_i & (p = 0, 1, 4, 5), \\ +\bar{\chi}^i \Gamma_{\mu_1 \ldots \mu_p} \Psi_i & (p = 2, 3). \end{cases}$$

B The geometry of the scalar manifold

The scalars of the theories studied in this paper parameterize the manifold:

$$\mathcal{M} \;=\; \frac{SO(5,n)}{SO(5) \times SO(n)} \times SO(1,1) \tag{B.1}$$

where the $SO(1,1)$ factor is spanned by the scalar σ of the supergravity multiplet and the rest by the scalars ϕ^x of the matter multiplets.

Having introduced the coset representatives $L_{\tilde{I}}{}^{A}$, one can determine the metric and curvatures of the manifold (B.1). Indeed the vielbeins are obtained from

$$f_x^{aij} = -2L^{\tilde{I}a}\partial_x L_{\tilde{I}}^{ij}, \tag{B.2}$$

whereas the metric is given by

$$f_x^{ija}\, f^{a}_{y\,ij} = 4g_{xy}. \tag{B.3}$$

The inverse of the vielbeins are defined via

$$f_x^{ija} f_{kl}^{x\,b} = 4\Big(\delta_k^{[i}\delta_l^{j]} - \frac{1}{4}\Omega^{ij}\Omega_{kl}\Big)\delta^{ab}. \tag{B.4}$$

As usual, the vielbeins are covariantly constant with respect to the full covariant derivative:

$$\partial_x f_y^{ija} - \Gamma_{xy}{}^{z} f_z^{ija} - Q_{x\,b}{}^{a} f_y^{ijb} - Q_{x\,k}{}^{i} f_y^{kja} - Q_{x\,k}{}^{j} f_y^{ika} = 0, \tag{B.5}$$

where Γ^z_{xy} denotes the Christoffel symbols on $\mathcal{M}$.

The $USp(4)$ and $SO(n)$ curvatures fulfill the following identities:

$$\begin{aligned} R_{xy\,i}{}^{j} &= -\frac{1}{4} f^{a}_{[x\,ik}\, f^{a\,kj}_{y]}, && \text{(B.6)}\\ R_{xy\,a}{}^{b} &= \frac{1}{4} f^{ij}_{[x\,a}\, f_{y]\,ij}{}^{b}, && \text{(B.7)} \end{aligned}$$

which can also be found as the solutions to the integrability conditions coming from the differential equations satisfied by the coset representatives

$$\begin{aligned} D_x L_{\tilde{I}ij} &= -\frac{1}{2} L^{a}_{\tilde{I}} f^{a}_{xij}, && \text{(B.8)}\\ D_x L^{\tilde{I}}_{ij} &= \frac{1}{2} L^{\tilde{I}\,a} f^{a}_{xij}, && \text{(B.9)}\\ D_x L^{a}_{\tilde{I}} &= -\frac{1}{2} f^{a}_{xij} L^{ij}_{\tilde{I}}, && \text{(B.10)}\\ D_x L^{\tilde{I}a} &= \frac{1}{2} f_x^{ija} L^{\tilde{I}}_{ij}, && \text{(B.11)} \end{aligned}$$

where D_x denotes the $USp(4)$ and $SO(n)$ covariant derivative. Let us finally display the useful identity:

$$L_{(\tilde{I}ik} L_{\tilde{J})}{}^{jk} = \frac{1}{4}\,\delta_i^j\, L_{\tilde{I}}{}^{kl} L_{\tilde{J}kl}, \tag{B.12}$$

which is nothing but the defining relation of the $SO(5)$ Clifford algebra used to convert the $SO(5)$ index of the $L_{\tilde{I}}{}^{A}$ into the corresponding composite $USp(4)$ index with the property that

$$L_{\tilde{I}}{}^{ij} = -L_{\tilde{I}}{}^{ji}\,, \qquad L_{\tilde{I}}{}^{ij}\,\Omega_{ij} = 0. \tag{B.13}$$

C Superspace analysis

In this appendix we want to propose a brief analysis of the constraints needed to reproduce the theory built in the main text in superspace. This will help us understand to which extent the couplings proposed here (besides non-minimal ones) are general.

In the superspace formalism, the dynamics of the fields is determined by the constraints one imposes on the supercurvatures. To classify the possible consistent sets of constraints, one adopts a general strategy which is based on group-theoretical analysis [48]. The superspace formalism gives quite stringent restrictions on the possible couplings of the various multiplets of the theory and the analysis of the lowest-dimensional components of the various superfields (supertorsion and supercurvatures) reveals what freedom one has in coupling for instance the gravity multiplet to the matter ones.

Moreover, we report some additional equalities, derived by the solution of the superspace Bianchi identities for the various supercurvatures, which are useful to derive some of the properties of the shifts in the transformation laws of the fermionic fields. The same relations could be derived at the component level by closing the supersymmetry algebra on the various fields.

We denote a generic superform as

$$\Phi = \frac{1}{n!} e^{A_1} \dots e^{A_n} \Phi_{A_n \dots A_1}, \tag{C.1}$$

where $A = (\mu, \alpha i)$ collectively denote vector (μ) and spinor (αi) indices, and e^A are the supervielbeins (which means that the projection of $e^{\alpha i}$ on ordinary space-time $dx^\mu \psi_\mu^{\alpha i}$ contains the gravitino field).

The (super-)torsion and (super-)curvatures are defined as

$$De^A = T^A, \qquad DT^A = e^B R_B{}^A, \tag{C.2}$$

$$F^I = dA^I + \frac{1}{2} g_S f^I_{JK} A^J A^K, \qquad G = da, \tag{C.3}$$

$$H^M = dB^M + g_A \Lambda^M_N \, a B^N, \tag{C.4}$$

and they satisfy the Bianchi identities:

$$DT^A = e^B R_B{}^A, \qquad DR_B{}^A = 0, \tag{C.5}$$

$$DF^I = 0, \qquad dG = 0, \tag{C.6}$$

$$DH^M = g_A \Lambda^M_N \, G B^N. \tag{C.7}$$

At this level, one tries to impose constraints on such superfields which are compatible with their Bianchi Identities (BI) such as to remove all the auxiliary fields that appear in their component expansion.

In the language of superspace, the basic constraint one has to impose to close $\mathcal{N}=4$ supersymmetry in a linear way is given by

$$T^m_{\alpha i\beta j} = \frac{1}{2}\Omega_{ij}\Gamma^m_{\alpha\beta}. \tag{C.8}$$

The other basic constraints on the torsion field are given by the definition of the $T_{\alpha im}{}^n$ and $T_{\alpha i\beta j}{}^{\gamma k}$ components. To analyze the freedom we have, and therefore the possibility of coupling gravity with matter, we make a group theoretical analysis of these structures.

The $T_{\alpha i\beta j}{}^{\gamma k}$ tensor contains the following $SO(5)_{USp(4)_R}$ representations $5\times\mathbf{4_4}+3\times\mathbf{4_{16}}+\mathbf{4_{20}}+3\times\mathbf{16_4}+2\times\mathbf{16_{16}}+\mathbf{16_{20}}+\mathbf{20_4}+\mathbf{20_{16}}+\mathbf{20_{20}}$, whereas $T_{\alpha im}{}^n$ contains $2\times\mathbf{4_4}+2\times\mathbf{16_4}+\mathbf{20_4}+\mathbf{40_4}$.

Using the connection redefinition $\omega_{\alpha i\, m}{}^n \to \omega_{\alpha i\, m}{}^n + X_{\alpha i\,[mp]}\eta^{pn}$ we are free to reabsorb many of the components of $T_{\alpha im}{}^n$ which then remains with only the $\mathbf{4_4}+\mathbf{16_4}+\mathbf{40_4}$ representations. Moreover, using the gravitino redefinition $\psi^{\alpha i}\to e^m H_m{}^{\alpha i}$ we are left with only the $\mathbf{40_4}$. This latter is then set to zero by the lowest dimensional T-BI, which is the equation $R_{(\alpha i\beta j\gamma k)}{}^m = 0$.

At the same time the $T_{\alpha i\beta j}{}^{\gamma k}$ tensor remains with the irreps $2\times\mathbf{4_4}+\mathbf{4_{16}}+\mathbf{4_{20}}$ which allows only for structures of the kind $\chi_{\alpha i}\delta^\gamma_\beta\Omega_{jk}$ and $Q_{\alpha i\,(jk)}\delta^\gamma_\beta$. Indeed the first kind of term is needed to close the BI of the graviphoton, whereas the second is used to couple matter (it has the same tensor structures as an $USp(4)$ connection, or more precisely its pull-back on superspace). This means that, even if one has not yet defined the scalar manifold, the type of interaction between the gravity sector and the matter must be mediated by the appearance of some type of $USp(4)$ connection field.

Once the connection-type term has been reabsorbed in the definition of a new supercovariant derivative and the F-BI (with coupling to matter) have been solved, one finds that the definition of the torsion constraint which is compatible with the supersymmetry transformations presented in this paper is given by

$$\begin{aligned} T_{\alpha i\beta j}{}^{\gamma k} &= \frac{i}{2\sqrt{3}}\Big(\chi_{\alpha i}\delta^{\gamma k}_{\beta j} + \chi_{\beta j}\delta^{\gamma k}_{\alpha i} + 2\chi^{\gamma k}C_{\alpha\beta}\Omega_{ij} + 4\chi^\gamma_{[i}\delta^k_{j]}C_{\alpha\beta} - \chi_{\alpha j}\delta^{\gamma k}_{\beta i} \\ &\qquad - \chi_{\beta i}\delta^{\gamma k}_{\alpha j} + \chi^k_\alpha\delta^\gamma_\beta\Omega_{ij} - \chi^k_\beta\delta^\gamma_\alpha\Omega_{ij}\Big), \end{aligned} \tag{C.9}$$

which translates in components as the following three-Fermi term in the transformation rule of the gravitino:

$$\begin{aligned} \delta_\varepsilon\psi^k_\mu &= \ldots + \frac{i}{2\sqrt{3}}\Big(\bar\psi^i_\mu\chi_i\varepsilon^k - \bar\varepsilon^i\chi_i\psi^k_\mu + 2\bar\psi^i_\mu\varepsilon_i\chi^k - 2\bar\psi^k_\mu\varepsilon^i\chi_i \\ &\qquad + 2\,\bar\psi^i_\mu\varepsilon^k\chi_i + \bar\varepsilon^k\chi_i\psi^i_\mu - \bar\psi^k_\mu\chi_i\varepsilon^i - \bar\varepsilon_i\chi^k\psi^i_\mu - \bar\psi^i_\mu\chi^k\varepsilon_i\Big), \end{aligned} \tag{C.10}$$

where in the dots are the components presented in the main text and the three-Fermi terms due to the pull-back of the $USp(4)$ connection.

For completeness, we list here also the other fundamental constraints:

$$\begin{aligned}
D_{\alpha i}\sigma &= -\frac{i}{2}\chi_{\alpha i}, &\text{(C.11)}\\
D_{\alpha i}\phi^x &= -\frac{i}{2}\lambda^{ja}_{\alpha} f^x_{ij\,a}, &\text{(C.12)}\\
G_{\alpha i\beta j} &= -\frac{i}{2\sqrt{2}}\Sigma^2\Omega_{ij}C_{\alpha\beta}, &\text{(C.13)}\\
F^I_{\alpha i\beta j} &= i\Sigma^{-1}L^I_{ij}C_{\alpha\beta}, &\text{(C.14)}\\
H^M_{\alpha i\beta j\gamma k} &= 0. &\text{(C.15)}
\end{aligned}$$

As promised, we also show the equalities that one derives between the various shifts in the supersymmetry laws of the Fermi fields by solving the superspace Bianchi identities (BI). From the H-BI we derive

$$(-4U+2S)_{[i}{}^k L^M_{j]k} - V^a_{[ij]}L^M_a + 2(S+U)_{[i}{}^k L^M_{j]k} = \Sigma^2\frac{1}{\sqrt{2}}\Lambda^M{}_N L^N_{ij}, \qquad \text{(C.16)}$$

whereas from the F-BI we get

$$(-4U+2S)_{[i}{}^k L^M_{j]k} + V^a_{[ij]}L^M_a + 2(S+U)_{[i}{}^k L^M_{j]k} = 0. \qquad \text{(C.17)}$$

Moreover, the closure of the ϕ-BI implies that

$$\begin{aligned}
V^a_{ij} &= \frac{1}{\sqrt{2}}\Sigma^2\,\Lambda^N{}_M L_{Nij}L^{Ma} &\text{(C.18)}\\
T^a_{ij} &= -\Sigma^{-1} f^I_{JK}\, L_{I(i}{}^m L^J_{|m|j)}L^{Ka}. &\text{(C.19)}
\end{aligned}$$

Using the solution of these quantities in terms of the coset representatives, the (C.17) equation becomes just $C^{IM}=0$ (for the Abelian part, an identity for the non-Abelian).

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